MULTIMEDIA UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EMT2046 – ENGINEERING MATHEMATICS 4

(All Sections / Groups)

1 MARCH 2019

3.00 PM – 5.00 PM

(2 Hours)

GENERAL INSTRUCTIONS:

1. This exam paper consists of **7 pages** (including cover page) with FOUR questions and an APPENDIX.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Consider the following linear programming (LP) problem:

$$\begin{aligned} \text{Maximize } z = {} & x_1 - x_2 + 2x_3 \\ \text{subject to: } & 3x_1 - x_2 + x_3 \le 4 \quad \text{(Constraint 1)} \\ & x_2 + 2x_3 \le 11 \quad \text{(Constraint 2)} \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

By introducing slack variables where appropriate, express the LP problem in standard form. Then solve the LP problem using the simplex method. Be sure to specify the entering and leaving variables, as well as the elementary row operations for each iteration. For the benefit of comparison, a partially-filled simplex tableau at the optimal stage is shown below:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	3	0	0	4/3	1/3	9
?	0	?	?	?	?	?	?
?	0	?	?	?	?	?	?

[15 marks]

(b) Convert the above LP problem to its dual.

[5 marks]

(c) Using appropriate properties, deduce the optimal solution and optimal value for the dual problem in (b).

[3 marks]

(d) What would happen to the optimal value of the primal problem if:

(i) the right-hand-side (RHS) of its Constraint 1 is increased by 1 unit?

(ii) the RHS of its Constraint 2 is decreased by 1 unit?

[2 marks]

Continued...

Question 2

(a) Consider the ordinary differential equation:

$$y'(x) = \frac{x}{y}$$
$$y(0) = 1$$

Using the Runge Kutta method of order four with step size $h = 0.1$, approximate $y(0.2)$.

[13 marks]

(b) The following system of linear equations is used to determine the concentration values, x_1, x_2, x_3 and x_4 of the chemicals in a system of coupled reactors.

$$\begin{aligned} x_1 \qquad\; + 5x_3 - 2x_4 &= -2 \\ 8x_1 - 2x_2 + 4x_3 \qquad\; &= 8 \\ 3x_1 + 6x_2 \qquad\; + \; x_4 &= 3 \\ 2x_1 \qquad\; + 2x_3 + 9x_4 &= 3 \end{aligned}$$

Find the first two iterations of the Gauss-Siedel method in solving the above linear system, using the initial values $\mathbf{x}^{(0)} = \mathbf{0}$. If necessary, rearrange the equations to achieve convergence of the approximated solutions.

[12 marks]

Continued...

Question 3

(a) Suppose X and Y are continuous random variables with joint probability density function(pdf):

$$f_{XY}(x,y)=\frac{1}{3}(kx+y)$$

for $0<x<2$ and $0<y<1$, where k is a constant.

(i) What is the value of k?

[4 marks]

(ii) Find the marginal pdf of X, $f_X(x)$.

[3 marks]

(iii) Find the marginal pdf of Y, $f_Y(y)$.

[3 marks]

(iv) Are X and Y statistically independent?

[2 marks]

(v) Find $P(1<X<2,\ 0.5<Y<1)$

[4 marks]

(b) Suppose X is a continuous random variable with probability density function:

$$f_X(x)=\frac{3x^2}{35}$$

for $-2<x<3$. What is the probability density function of $Y=X^2$?

[9 marks]

Continued...

Question 4

Consider a particle moving among the states 0, 1 and 2. The particle's movement is described by a Markov chain with the following one-step transition matrix:

$$\begin{bmatrix} a & b & c \\ 0.3 & 0.1 & a \\ b & 0 & a \end{bmatrix}$$

(a) Find the values of a, b and c.

[3 marks]

(b) Draw the state transition diagram.

[4 marks]

(c) Find the two-step transition probability matrix, and then interpret $P_{10}^{(2)}$.

[5 marks]

(d) Suppose that at $t = 0$, the particle cannot be at state 2, but it may be at states 0 or 1 with equal probability. Find the probability that the particle is at state 0 after two transitions.

[4 marks]

(e) Specify the classes of the Markov chain. Is the Markov chain irreducible?

[2 marks]

(f) Find the period of all the states.

[2 marks]

(g) Find the state probabilities after a long run.

[5 marks]

Continued...

APPENDIX

TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with $L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$.

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term $\left|\frac{h^2}{6} f^{(3)}(c_x)\right|$.

8. Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}(f(a)+f(b)) - \frac{h^3 f''(\xi)}{12}$$

for some ξ in (a, b) and $h = b - a$.

9. Simpson's rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{h^5}{90} f^{(iv)}(\xi)$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$.

10. Newton-Raphson's method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1,2,\ldots$$

11. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2} Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

12. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

13. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf\left(x_{i+1}, y_i + k_3\right),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

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